give an anesthetic or a poison by way of the mouth which is almost impossible. However, substances can be introduced into the alimentary tract through the anus and the desired results obtained.

Such is the method used in this laboratory. Chloroform is injected into the cloaca and a string tied in front of the anus to prevent the ejection of the liquid. Five c.c. of chloroform thus given will anesthetize an eight-inch turtle sufficiently for dissection in thirty to forty-five minutes.

The value of this method is threefold. First, a string and a pipette constitute the necessary equipment; second, the ease with which the anesthetic can be given is evident; and third, there is no danger of the specimens coming out from under the chloroform.

Newton Miller
University of Utah

SCIENTIFIC BOOKS

Combinatory Analysis. By Major Percy A. MacMahon. Cambridge University Press. 1915, Vol. 1, xix + 300 pp., and 1916, Vol. 2, ix + 340 pp.

One of the four grand divisions of what may be called properly static mathematics is the theory of configurations. It includes the construction out of given elements of compound forms under certain given conditions or restrictions; together with the characters possessed by such constructions when they are varied under given laws, such as, for instance, the character of transitivity, or that of primitivity; and the laws of dependence of such constructions upon each other; as well as finally the invention of new or ideal elements of mathematics that enable the solution of problems of construction to be effected. These constructions vary from the mere permutation of a linear series of elements up to the complicated trees of chemical combinations studied by Cayley, and in general to all sorts of problems in what has been happily denominated tactics by Cayley, or syntactics by Cournot. We find in its field the construction of magic squares, of Latin squares, of Latin-Greek squares, of triangles, stars, polygons, chess problems, routes over net works, problems of topography, and without much stretch of imagination we might now include the disposition of the elements of war. The field is obviously large in extent, and in a wide variety of aspects fascinating. From certain points of view one might be tempted to conclude that we could include in it all mathematics, for the definition given by C. S. Peirce made mathematics the science of ideal constructions and their applicability to the world as it is.

The study of configurations usually begins with combinatory analysis. By this is usually meant the study of the arrangements along a line of a collection of objects, either as individuals or in groups; arrangements at the nodes of a lattice; combinations of arrangements. Such problems arise not only as matters of tactic, curious problems or puzzles, but in the determination of the number of such arrangements needed in solving problems in the theory of probabilities.

The treatise of Professor MacMahon undertakes to present some very general methods of handling such studies. These methods consist for a large part in the construction of enumerating generating functions, and involve considerable study of symmetric functions and certain differentiating operators. In the course of this study he arrives at some very elegant theorems. These methods not only enumerate the possible forms, but in many cases afford methods of actual construction of the entire list of such possible forms. They are very powerful and have enabled the author to solve problems that were considered for a long time to be beyond the reach of mathematical analysis. His success and presentation in complete form may induce others to study this important branch of mathematics.

There are eleven sections, and the topics under consideration will give some idea of the character of the treatise. Section one considers ordinary symmetric functions and their connection with the theory of distribution of objects into parcels. The operators which are useful for these purposes are developed, and their algebra considered, turning out to be quite analogous to the algebra of symmetric

functions. A distinction is drawn between the parcel of objects, in which the order of arrangement in the parcel is immaterial, and the group of objects, in which the order of arrangement in the group is material. For instance if we sort 3 α's 2 β's, 1 γ, 1 δ into seven boxes of which four boxes are exactly alike, and three boxes alike but different from the first four, we find that we have a problem of distribution of objects of type (321^2) into parcels of type (43), which can be done in 11 ways. This number 11 may be found by a distribution function, derived from the theory of symmetric functions. This function gives, for instance, for the various types of 4 objects distributed into parcels of type (2) these results: for type (4), 2 ways, for type (31) 3 ways, for type (22) 4 ways, for type (211) 5 ways, for type (1111) 7 ways. If, however, the distribution is into groups rather than into parcels, we have for type (4) 2 ways, for type (31) 6 ways, for type (22) 10 ways, for type (211) 18 ways, and for type (1111) 36 ways. The determination by the function consists in finding the coefficients in formulæ that arise from the theory of symmetric functions. These coëfficients may be found directly for the individual terms by using the operators referred to.

Section two considers the theory of separations, a separation being a distribution of the numbers constituting a partition of some integer into parcels, or groups. Extensive generalizations are possible from the formulæ and the operators produced. The application to sets of objects of given types and their distributions resolves more complicated problems than those given before. For instance, with a set of four threefold objects, $a_1a_2a_3$, $a_1a_2a_3$, $b_1b_2b_3$, $c_1b_2c_3$ can be formed 38 cases of distribution into the types (211), (22), (211), namely the objects $a_1a_1b_1c_1$, $a_2a_2b_2b_2$, $a_3a_3b_3c_3$, and the different permutations of these arrangements.

Section three deals with permutations, particularly with points useful in the general theory of combinations and distributions. A certain master theorem is deduced which has great resolving power. In particular it solves the problem of ascertaining the number of permutations in which every letter occupies a new place, and in expressing sums of powers of binomial coefficients. The notion of lattice permutation is introduced, by which is meant that if any permutation be made of a α's, b β's, c γ's, etc., to be a lattice permutation it must be such that reading it from left to right, at no point of it will the number of α's so far written be less than the number of β's, nor number of β's less than the number of γ's, etc. For instance, for 2 α's and 2 β's the lattice permutations are $\alpha\alpha\beta\beta$, and $\alpha\beta\alpha\beta$. The permutation $\alpha\beta\beta\alpha$ is not a lattice permutation because when we arrive at the third letter, we shall have 2 β's and only 1 α. These are called lattice permutations because they serve to handle arrangements of integers at the nodes of a rectangular lattice in a plane, or in space.

Section four considers compositions of integers, by which is meant the permutations of the partitions of the integer. In connection with these some new symmetric functions are introduced. An application to Newcomb's problem is made. It is this: given p cards marked 1, q marked 2, r marked 3, etc., which are shuffled and dealt in such wise, that as long as a card is not of lower number than the preceding it is placed upon the preceding, but if lower it must start a new pile; what is the probability that there are at most m piles when all have been dealt?

Section five introduces the notion of perfect partition, that is partitions such that each contains only one partition of each lower number. For instance, for 7, a perfect partition is (4111), since we have only one partition of 1, 2, 3, 4, 5, 6. These are then applied to distributions upon a chess board. A connection is thus arrived at between magic squares and the general theory. The enumeration of Latin squares is effected by generating functions, thus solving a long-standing problem. For instance, the number of reduced Latin squares of order 1, is 1, of order 2, is 1, of order 3, is 1, of order 4, is 4, and of order 5, is 52.

Section six enumerates the partitions of multipartite numbers. A multipartite num-

ber is one that would be called in general algebra a multiplex, as (5, 3) or (3, 6, 2, 7).

Several tables of homogeneous functions, distribution functions, and enumerations close the first volume.

Section seven is devoted to the algebraic side of the partition of numbers, giving in some detail the present state of the theory, but omitting the purely arithmetic side. There are given here further developments connected with symmetric functions.

Section eight considers the theory of partitions as based upon Diophantine inequalities, generalizing the whole treatment. A chapter is devoted to the further study by this means of magic squares, the object being their enumeration rather than construction.

Sections nine and ten study partitions in two dimensions, including a complete solution of another long-standing problem. The problem of three-dimensional partitions relating to a cubic lattice is also attacked.

Section eleven relates to symmetric functions of several systems of quantities with applications to distribution functions.

The second volume closes with tables of symmetric functions of two systems, and enumeration of solid graphs.

If one were to undertake to characterize the treatise of Professor MacMahon briefly he would probably best state its field by saying that it is a development of the algebra of symmetric functions with applicatin to various generating expressions whose coefficients find use in enumeration problems of distribution. Professor MacMahon has occupied himself with the development of this theory for some years and the treatise is a systematic presentation of his results. No brief account can be given of the very skilful methods employed. It shows amply that alongside of the alternating functions so long studied in determinant forms, the symmetric functions are equally important and have their field of application. It exemplifies how different branches of mathematics can be correlated so as to be useful in reducing problems. It also draws strongly attention to the fact that there still remains in the field of algebraic form plenty of opportunity for the interested student to do research work of high order. Indeed it would seem that courses on symmetric functions at least should be offered alongside of other courses in algebra, such as theory of equations, determinants, groups, and the like. The whole theory of the construction of algebraic forms for certain specific purposes has been enriched here with a valuable contribution.

JAMES BYRNIE SHAW

SPECIAL ARTICLES

INHERITANCE OF OIL IN COTTON

The table of oil percentages given below suggests the possibility of producing divergent strains or biotypes from a "variety" of cotton, the one having seeds relatively high in oil content, the other relatively low in oil content.

The top line of figures gives the analysis (ether extract) of the seed from several mother plants, followed in column by the analysis of the seed of three of their progeny plants, respectively.

	Parent						
	17.33	20.64	16.58	18.97	16.79	18.92	16.87
Progeny.....	18.21	22.00	17.16	22.10	17.75	19.37	18.47
	18.67	20.82	18.40	21.17	17.86	19.45	18.92
	19.13	21.06	17.86	21.36	17.52	19.19	18.40

The three "high" parents have an average of 19.51 per cent. oil, and their nine progeny plants an average of 20.72 per cent. oil.

The four "low" parents have an average of 16.89 per cent. oil, and their twelve progeny plants an average of 18.20 per cent. oil.

The maximum difference between parents is 4.06 per cent. oil, and the maximum difference between plants of the progeny generation is 4.94 per cent. oil.

A seasonal variation raising the oil content of all plants in the progeny year is noted.

A later report will give the correlation between oil content of the seed and other characters.

E. P. HUMBERT
AGRICULTURAL EXPERIMENT STATION,
COLLEGE STATION, TEXAS